on an ogive-cylinder body with cold wall conditions," NACA RM A56B15 (1956).

[38] Goebel, T. P., "Transfer of M. I. T. yawed cone data from wind to body axes," North American Rept., Appendix A, NA-60-952 (July 1960).

[39] Goebel, T. P., "Computation of *Ap*, *Bp*, *Dp* coefficients using Cheng's hypersonic shock layer theory," North American Rept., Appendix D, NA-60-952 (July 1960).

[40] Goebel, T. P., "Correlation of cold-flow, yawed-cone pressure data," North American Rept., Appendix G, NA-60-952 (July 1960).

[41] Goebel, T. P., "Computation of *Ap*, *Bp*, *Dp* coefficients using Newtonian concepts," North American Rept., Appendix E, NA-60-952 (July 1960).

MARCH 1963 AIAA JOURNAL VOL. 1, NO. 3

Nonequilibrium Flow Past a Wedge

R. Capiaux* and M. Washington*
Lockheed Missiles and Space Company, Palo Alto, Calif.

An exact numerical solution is obtained for the chemically reacting flow past a wedge. The freestream is either in equilibrium or out of equilibrium but nonreacting. The attached shock wave is shown to be either concave, convex, or straight, depending on the values of the amount of dissociation in the freestream and a parameter describing the amount of energy contained in the freestream relative to the gas dissociation energy. Numerical examples are presented illustrating these regimes. The flow field is characterized by the presence of an entropy layer and a relaxation layer, both easily identifiable in the presentation of the numerical results.

Introduction

THE equations of motion of a compressible inviscid fluid undergoing chemical reactions and internal relaxation effects have been derived by Wood and Kirkwood.[1] In particular, the authors transform the one-dimensional unsteady flow into a characteristic form and find that the "frozen" speed of sound plays a role analogous to the sound velocity in the case of a nonreactive fluid. Chu[2] arrives at the same conclusion and obtains the characteristic equation for steady two-dimensional and axially symmetric flows.

One of the important differences between a chemically reacting gas flow and a nonreacting or equilibrium flow consists in an entropy production appearing in the energy equation. The entropy production term is a direct result of reactions in the flow field which are thermodynamically irreversible processes.

As Clarke[3] points out, the entropy production is not confined to the interior of boundary layers or shock waves because it is not associated with a transport phenomenon. Its influence therefore can spread over the entire flow field and will not depend explicitly on the gradients of velocity, temperature, and concentration.

In the present paper, a numerical solution is obtained for the chemically reacting, steady, two-dimensional flow past a wedge. This solution is the counterpart of Sedney's solution,[4] which treats the case of the flow of a vibrationally relaxing gas past a wedge. Viscosity, thermal conduction, and diffusion have been neglected purposely to emphasize the forementioned effects. A simple body configuration is selected in order to allow a direct comparison of the results with the simple results associated with a nonreacting fluid. The analysis is applied to a pure diatomic gas, such as oxygen or nitrogen, which obeys the model proposed by Lighthill[5] in its equilibrium dissociated state. The chemistry rate is described appropriately by a somewhat simplified but accurate rate equation introduced by Freeman.[6] The undisturbed fluid is allowed to assume either an equilibrium dissociated state or a nonreacting frozen state. Thus, the degree of dissociation of the freestream determines whether the flow in the wake of the leading shock will undergo a dissociation or recombination process.

The solution previously presented by Vincenti[7] neglects, as a consequence of the linearization assumption, the entropy production term; the flow therefore remains irrotational. In this presentation, the introduction of a streamline oriented coordinate system allows naturally for the accounting of the dissociation and recombination processes and the entropy increase associated with them. The equations of motion then are solved numerically by the method of characteristics.

Two-Dimensional Steady Flow: General Equations

The equations of motion will be derived in this section for a general, diatomic, reacting gas. Specialization to a particular equation of state and rate equation will be introduced in a later section. The geometry of the problem is presented in Fig. 1.

For the case of the two-dimensional flow in Cartesian coordinates, the equations of continuity and of momentum can be written as follows:

$$[\partial(\rho u)/\partial x] + [\partial(\rho v)/\partial y] = 0 \tag{1}$$

$$u(\partial u/\partial x) + v(\partial u/\partial y) = -1/\rho(\partial p/\partial x) \tag{2}$$

$$u(\partial v/\partial x) + v(\partial v/\partial y) = -1/\rho(\partial p/\partial y) \tag{3}$$

Here ρ is the mixture density, p the pressure, and u and v components of the velocity vector V on the x and y axes, respectively.

The energy equation can be written in several equivalent forms. If h is the enthalpy (including the chemical bond energy), t is the time, and D/Dt designates the convective derivative, the following relations hold:

$$\rho(Dh/Dt) - (Dp/Dt) = 0 \tag{4}$$

$$(D/Dt)[h + (V^2/2)] - (1/\rho)(\partial p/\partial t) = 0 \tag{5}$$

For the assumed nonequilibrium situation, the continuity

Presented at the IAS National Summer Meeting, Los Angeles, Calif., June 19–22, 1962; revision received December 17, 1962.

* Member, Fluid Mechanics, Mechanical and Mathematical Sciences Laboratory, Research Laboratories.

equation for the atomic species of mass fraction α can be written as follows:

$$D\alpha/Dt = \chi(p,\rho,\alpha)/\theta(p,\rho,\alpha) \tag{6}$$

The case of infinitely slow reaction (frozen flow) corresponds to $\theta(p,\rho,\alpha) \to \infty$, whereas the equilibrium flow conditions correspond to $\chi\ (p,\rho,\alpha)$ and $\theta(p,\rho,\alpha)$ tending simultaneously towards zero while the value of $D\alpha/Dt$ remains finite. In this latter case, α tends toward its equilibrium value designated by c, since $\chi \to 0$.

The nonequilibrium chemical state of the gas is further defined by two equations of state which are taken in the form

$$p = p(\rho,S,\alpha) \tag{7}$$

$$h = h(\rho,T,\alpha) \tag{7'}$$

The second law of thermodynamics can be written simply as

$$TdS = dh - 1/\rho\, dp - (\mu_1 - \mu_2)\, d\alpha \tag{8}$$

where μ_1 and μ_2 are the chemical potentials of the atoms and molecules, respectively.

From a combination of Eq. (4) and Eq. (8), it follows that the energy equation also can be expressed as

$$T(DS/Dt) = -(\mu_1 - \mu_2)(D\alpha/Dt) \tag{9}$$

For steady flows, integration of Eq. (5) yields

$$h + (V^2/2) = \text{const} = h_0 \tag{10}$$

Thus, for nonequilibrium flows, the total enthalpy remains constant along streamlines. Clarke[8] has shown that the difference in the chemical potentials can be expressed as

$$\mu_1 - \mu_2 = -RT \ln\left(\frac{c^2}{1-c}\,\frac{1-\alpha}{\alpha^2}\right) \tag{11}$$

where R is the universal gas constant divided by the molecular weight of the undissociated gas. The variable c is, as indicated in the foregoing, the atomic equilibrium concentration evaluated at the local p and T conditions.

The system of Eqs. (1–3, 6–9, and 11) constitutes a set of nine equations for the nine unknowns and therefore is sufficient, with application of appropriate boundary conditions, for a solution of the problem. The energy equation, Eq. (9), brings to light the important aspect of a flow involving chemical reactions. Since the reactions are thermodynamically irreversible processes, they lead to a convective entropy generation. It would seem logical, therefore, to change from a Cartesian coordinate system x, y to a coordinate system where this phenomenon can be studied more easily and also computed more accurately. This can be accomplished best by introducing the stream function ψ, which becomes a new independent variable. The physical coordinate y can, for instance, be chosen as the other independent variable.

The introduction of ψ as an independent variable for two-dimensional steady flow is similar to the expression of the one-dimensional unsteady equations of motion as functions of a Lagrangian coordinate. A ψ number identifies a particular streamline, thereby allowing one to follow the particle along its path. The ψ function is defined in the ordinary manner:

$$\rho u = \partial\psi/\partial y \tag{12}$$

$$\rho v = -\partial\psi/\partial x \tag{13}$$

It also follows, from the change of coordinates, that

$$\left(\frac{\partial}{\partial x}\right)_y = \left(\frac{\partial}{\partial y}\right)_\psi \frac{\partial y}{\partial x} + \left(\frac{\partial}{\partial x}\right)_y \frac{\partial \psi}{\partial x} = -\rho v\,\frac{\partial}{\partial \psi}$$

$$\left(\frac{\partial}{\partial y}\right)_x = \left(\frac{\partial}{\partial y}\right)_\psi + \left(\frac{\partial}{\partial \psi}\right)_y \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y} + \rho u\,\frac{\partial}{\partial \psi}$$

The convective derivative $D/Dt = u(\partial/\partial x) + v(\partial/\partial y)$ becomes, in the new coordinates,

$$D/Dt = v(\partial/\partial y) \tag{14}$$

The continuity equation is satisfied identically by the introduction of ψ. The two momentum equations (2) and (3) become

$$\partial u/\partial y = \partial p/\partial \psi \tag{15}$$

$$v(\partial v/\partial y) = -(1/\rho)(\partial p/\partial y) - u(\partial p/\partial \psi) \tag{16}$$

The rate and energy equations (6) and (9) become

$$v(\partial \alpha/\partial y) = \chi/\theta \tag{17}$$

$$v[\partial(S/R)/\partial y] = -[(\mu_1 - \mu_2)/RT]\, v(\partial\alpha/\partial y) \tag{18}$$

By manipulation of Eqs. (12) and (13), the following two equations can be obtained:

$$\partial x/\partial y = u/v \qquad \partial x/\partial \psi = -(1/\rho v)$$

where x is now a dependent variable.

By differentiating the first equation with respect to ψ and the second with respect to y and then equating them, the following relationship is obtained:

$$v\frac{\partial u}{\partial \psi} - u\frac{\partial v}{\partial \psi} - \frac{v}{\rho^2}\frac{\partial \rho}{\partial y} - \frac{1}{\rho}\frac{\partial v}{\partial y} = 0 \tag{19}$$

The new set of equations (15–19) and the unmodified set (7–8 and 11) define the problem.

Since the transformation is not conformal, the correspondence of angles from the (x, y) plane to the (ψ, y) plane must be written:

$$\frac{d\psi}{dy} = \rho v\left(\frac{u}{v} - \frac{1}{dy/dx}\right) \tag{20}$$

In order to find the characteristic curves and the differential equation valid along these lines, it first is desirable to eliminate the pressure from Eqs. (15) and (16) by using the differential form of the equation of state:

$$dp = (\partial p/\partial \rho)_{S,\alpha}\, d\rho + (\partial p/\partial S)_{\rho,\alpha}\, dS + (\partial p/\partial \alpha)_{\rho,S}\, d\alpha$$

Using the modified equations (15) and (16) as well as Eqs. (17–19), the following array of the simultaneous differential equations is obtained:

$$\begin{gathered}
\frac{\partial u}{\partial y} - A\frac{\partial \rho}{\partial \psi} - B\frac{\partial s}{\partial \psi} - F\frac{\partial \alpha}{\partial \psi} = 0 \\
v\frac{\partial v}{\partial \psi} + Au\frac{\partial \rho}{\partial \psi} + \frac{A}{\rho}\frac{\partial \rho}{\partial y} + Bu\frac{\partial S}{\partial \psi} + \frac{B}{\rho}\frac{\partial S}{\partial \psi} + Fu\frac{\partial \alpha}{\partial \psi} + \frac{F}{\rho}\frac{\partial \alpha}{\partial y} = 0 \\
v\frac{\partial u}{\partial \psi} - u\frac{\partial v}{\partial \psi} - \frac{v}{\rho^2}\frac{\partial \rho}{\partial y} - \frac{1}{\rho}\frac{\partial v}{\partial y} = 0 \\
v\frac{\partial S}{\partial y} = J\frac{\chi}{\theta} \qquad v\frac{\partial \alpha}{\partial y} = \frac{\chi}{\theta} \\
\frac{\partial u}{\partial \psi}d\psi + \frac{\partial u}{\partial y}dy = du \\
\frac{\partial v}{\partial \psi}d\psi + \frac{\partial v}{\partial y}dy = dv \\
\frac{d\rho}{\partial \psi}d\psi + \frac{\partial \rho}{\partial y}dy = d\rho \\
\frac{\partial S}{\partial \psi}d\psi + \frac{\partial S}{\partial y}dy = dS \\
\frac{\partial \alpha}{\partial \psi}d\psi + \frac{\partial \alpha}{\partial y}dy = d\alpha
\end{gathered} \tag{21}$$

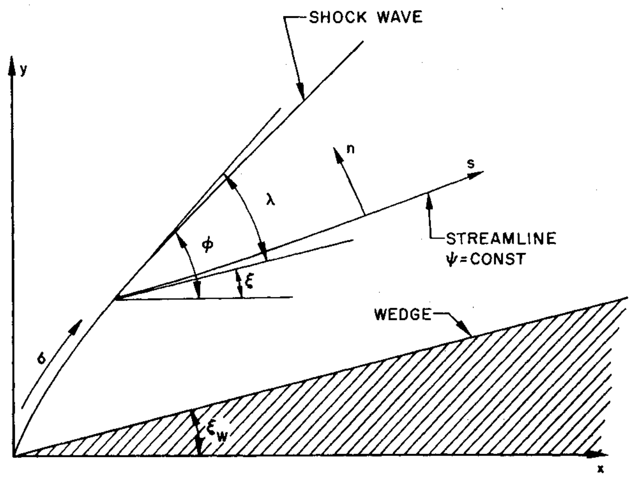

Fig. 1 Geometry of the physical problem

where

$$A = \left(\frac{\partial p}{\partial \rho}\right)_{S,\alpha} \qquad B = \left(\frac{\partial p}{\partial S}\right)_{\rho,\alpha} \qquad F = \left(\frac{\partial p}{\partial \alpha}\right)_{\rho,S}$$

and

$$J = -(\mu_1 - \mu_2)/RT$$

The characteristic curves can be defined as curves on which the derivatives of the fluid properties are indeterminate and therefore may undergo arbitrary discontinuities. For example, the condition that $\partial u/\partial \psi$ be indeterminate can be obtained by equating to zero both the numerator and denominator of the expression obtained for $\partial u/\partial \psi$. The numerator yields the direction of the characteristic curves as follows:

$$\frac{dy}{d\psi} = \frac{1}{\rho}\frac{a_f^2 - v^2}{a_f^2 u \pm a_f v(u^2 + v^2 - a_f^2)^{1/2}} \tag{22}$$

and

$$d\psi/dy = 0 \tag{23}$$

where

$$a_f^2 = (\partial p/\partial \rho)_{S,\alpha}$$

which is, by definition, the frozen speed of sound.

Equation (22), once transformed in dy/dx terms in the physical plane, yields the well-known result that the characteristic curves are inclined at an angle $\pm \sin^{-1}(a_f/V)$ to the velocity vector. Equation (23) reveals that the third characteristic is the streamline. The denominator of the expression for $\partial u/\partial \psi$ gives

$$\left\{A\left(u\dot{y} - \frac{1}{\rho}\right)^2 - \frac{v^2}{\rho^2}\right\}du + \left\{Au\left(u\dot{y} - \frac{1}{\rho}\right)\dot{y}^2 - A\left(u\dot{y} - \frac{1}{\rho}\right)^2 - \frac{v^2}{\rho^2}\right\}dp + A\,uv\dot{y}^2\,dv - \frac{v^2}{\rho^2}\left(\frac{B}{v}\,J\,\frac{\chi}{\theta} + \frac{F}{v}\,\frac{\chi}{\theta}\right)\dot{y}\,dy = 0$$

where $\dot{y} = dy/d\psi$. After inserting the value of $\dot{y}$ from Eq. (22), the following result can be obtained:

$$d\xi \pm \frac{(M_f^2 - 1)^{1/2}}{\rho V^2}\,dp - \frac{BJ(\chi/\theta) + F(\chi/\theta)}{a_f^2\rho^2 V^2}\,d\psi = 0 \tag{24}$$

where the angle ξ has been introduced and is defined as $\sin\xi = v/V$ or its equivalent $\cos\xi = u/V$. By introducing the angle ξ, the characteristic direction can be rewritten, and Eq. (22) becomes

$$dy/d\psi = (1/\rho V)[\cos\xi \pm \sin\xi(M_f^2 - 1)^{1/2}]$$

M_f is usual defined as the local frozen Mach number:

$$M_f = V/a_f$$

Shock Wave Curvature, Pressure Gradient on Wedge Surface

The shock wave curvature and the initial pressure gradient at the tip of the wedge surface can be obtained very simply. These two quantities are the initial values needed to start a numerical integration. The approach taken here is similar to the one introduced by Sedney[9] for a vibrationally relaxing gas and by Hsu[10] for a dissociating gas. If σ is the distance measured along the shock wave, s and n the distances measured along the streamlines and normal to the streamlines, respectively, then it follows that

$$\partial/\partial\sigma = \cos\lambda(\partial/\partial s) + \sin\lambda(\partial/\partial n)$$

where $\lambda = \varphi - \xi_w$($\varphi$ is the shock angle; ξ_w is the wedge angle)

$$\partial/\partial\sigma = (\partial\varphi/\partial\sigma)(\partial/\partial\varphi)$$

$\partial\varphi/\partial\sigma$ is the shock curvature designated as K_S. It follows then that

$$\frac{\partial}{\partial n} = \left(K_S\frac{\partial}{\partial\varphi} - \cos\lambda\frac{\partial}{\partial s}\right)\frac{1}{\sin\lambda} \tag{25}$$

Using the equations of motion (1–3, 6, and 9) and the equation of state in its differential form, it easily is shown that Eqs. (1–3) can be written in the intrinsic coordinate system (n, s) as follows:

$$\frac{M_f^2 - 1}{\rho V^2}\frac{\partial p}{\partial s} + \frac{\partial\xi}{\partial n} - \frac{1}{\rho a_f^2 V}\left(BJ\frac{\chi}{\theta} + F\frac{\chi}{\theta}\right) = 0$$
$$\frac{1}{\rho V^2}\frac{\partial p}{\partial n} + \frac{\partial\xi}{\partial s} = 0 \tag{26}$$

Using Eq. (25) and solving (26) for $\partial p/\partial s$ and $\partial\xi/\partial s$, one finds that the following expressions result:

$$\frac{\partial p}{\partial s}\left(\frac{M_f^2 - 1}{\rho V^2} - \frac{\cot^2\lambda}{\rho V^2}\right) = \frac{1}{\rho a_f^2 V}\left(BJ\frac{\chi}{\theta} + F\frac{\chi}{\theta}\right) - \frac{\partial\xi}{\partial\varphi}\frac{1}{\sin\lambda}K_S - \frac{\cot\lambda}{\sin\lambda}\frac{K_S}{\rho V^2}\frac{\partial p}{\partial\varphi} \tag{27}$$

$$\frac{\partial\xi}{\partial s}(M_f^2 - 1 - \cot^2\lambda) = \frac{\cot\lambda}{\rho a_f^2 V}\left(BJ\frac{\chi}{\theta} + F\frac{\chi}{\theta}\right) - \frac{\partial\xi}{\partial\varphi}\frac{\cot\lambda}{\sin\lambda}K_S - \frac{M_f^2 - 1}{\rho V^2}K_S\frac{\partial p}{\partial\varphi}\frac{1}{\sin\lambda} \tag{28}$$

If $(\partial\xi/\partial s)_w$, the curvature of the wedge, is known, Eqs. (27) and (28) can be solved for K_S, the initial shock curvature, and $\partial p/\partial s$, the pressure gradient at the tip of the wedge.

For the particular case of a straight wedge, $(\partial\xi/\partial s)_w = 0$ and

$$\frac{\partial p}{\partial s} = \frac{1}{\rho a_f^2 V}\left(BJ\frac{\chi}{\theta} + F\frac{\chi}{\theta}\right)\frac{\partial p/\partial\varphi(1/\cos\lambda)}{(\partial\xi/\partial\varphi)(1/\sin\lambda) + [(M_f^2 - 1)/\rho V^2](1/\cos\lambda)(\partial p/\partial\varphi)} \tag{29}$$

The functions $\partial p/\partial\varphi$ and $\partial\xi/\partial\varphi$ are functions of the free-stream conditions only and can be evaluated directly from the conservation equations across the shock.

Equation (28) gives the initial shock curvature for a straight wedge as

$$K_S = \frac{[BJ(\chi/\theta) + F(\chi/\theta)](1/\rho a_f^2 V)}{(\partial\xi/\partial\varphi)(1/\sin\lambda) + [(M_f^2 - 1)/\rho V^2](1/\cos\lambda)(\partial p/\partial\varphi)} \tag{30}$$

Boundary Conditions

The boundary conditions are to be applied at the wedge surface and at the shock wave surface. Designating the wedge angle by ξ_w, it follows directly that $\sin\xi_w = (v/V)$ wedge surface, and $\cos\xi_w = (u/V)$ wedge surface. The shock wave surface is taken as an infinitesimally thin surface across which discontinuities of certain physical quantities can take place. However, no dissociation or recombination occurs across this wave surface so that α assumes, immediately behind the shock front, the same value it had ahead of it. A sudden jump in temperature corresponds, therefore, to an infinitely fast achievement of thermodynamic equilibrium of translational, rotational, or vibrational degrees of freedom. The attainment of equilibrium between these classes and the dissociation, or recombination process, occur, therefore, entirely behind the shock wave front.

The jump conditions across the shock wave are governed by the three usual conservation equations. Denoting by subscript 0 the conditions ahead and by 1 the conditions immediately behind, it follows that

$$\begin{aligned} \rho_0 V_0 \sin\varphi &= \rho_1 V_{N_1} \\ p_0 + \rho_0 V_0^2 \sin^2\varphi &= p_1 + \rho_1 V_{N_1}^2 \\ V_0 \cos\varphi &= V_{T_1} \\ h_0 + \tfrac{1}{2}V_0^2 &= h_1 + \tfrac{1}{2}V_1^2 \end{aligned} \tag{31}$$

where φ is the shock angle and V_T and V_N are the components of the velocity vector V tangent and normal to the shock wave. Equations (31) in conjunction with the equations of state (7) and (7′) can be solved for p_1, ρ_1, V_1, and h_1.

Specialization to the Ideal Dissociating Gas

In order to obtain some simplification of the equations describing the chemistry of the gas, it will be assumed that the diatomic gas under consideration is an ideal dissociating gas as introduced by Lighthill.[5] In this case, the equilibrium proportion by mass of atoms in the mixture or the equilibrium mass fraction is given by

$$c^2/(1 - c) = (\rho_D/\rho_e)e^{-D/RT} \tag{32}$$

where D is the dissociation energy per unit mass and ρ_D is a characteristic density, which, as indicated by Lighthill, can be taken as a constant over the range of temperature from 1000° to 7000°K. R, the gas constant, was defined in Eq. (10); ρ has been given the subscript e to indicate that this is the equilibrium density for the local pressure and temperature conditions; ρ_D is typically 150 g/cm³ for oxygen and 130 g/cm³ for nitrogen; D has a value of $5.9 \times 8.32 \times 10^{11}/32$ erg/g, i.e., cm²/sec², for oxygen and $11.3 \times 8.32 \times 10^{11}/28$ for nitrogen. Figure 2 is a graphical representation of Eq. (32).

The thermal equation of state of the mixture is taken as

$$p = \rho RT(1 + \alpha) \tag{33}$$

and the caloric equation of state as

$$h = (4 + \alpha)RT + D\alpha \tag{34}$$

thereby defining Eqs. (7) and (7′). As Freeman[6] points out, the ideal dissociated gas has only one half of its vibrational energy excited, and thus the ratio of specific heats is $\frac{4}{3}$. In other words, the contribution from the various degrees of freedom to the energy is the same for atoms and molecules. The atoms have three degrees of freedom, whereas the molecules have six, but their mass is twice as large. The internal energy is then

$$e = 3RT + D\alpha \tag{34′}$$

where $D\alpha$ is the energy absorbed by dissociation. The rate equation is also taken from Freeman and is written as

$$d\alpha/dt = C\rho T^{-n}[(1 - \alpha)e^{-D/RT} - \rho/\rho_D\alpha^2] \tag{35}$$

where C is a constant with a dimension of [(temperature)^n/(density)(time)]. Comparison of Eq. (35) with the general diatomic rate equation, given by Clarke,[11] indicates the close similarity between the two equations. Clarke gives

$$\frac{d(\rho\alpha)}{dt} = \left(\frac{p}{RT}\right)^3 \frac{1}{W_m^2} K_R\left[-\frac{4\alpha^2}{(1+\alpha)^2} + K_x\frac{1-\alpha}{1+\alpha}\right]$$

where K_R is the recombination rate expressed in cm⁶/mol-sec; W_m is the molecular weight of the diatomic gas, K_x the equilibrium constant in terms of mass fractions, and ρ_e the equilibrium density. Since $K_x = 4c^2/(1 - c^2)$ and the relation between ρ_e and ρ is $\rho_e/\rho = (1 + \alpha)/(1 + c)$ at the same ρ and T, it follows that

$$\frac{d\alpha}{dt} = 4K_R\frac{\rho^2}{W_m^2}(1+\alpha)\left[-\alpha^2 + \frac{c^2}{1-c^2}(1-\alpha^2)\right] \tag{36}$$

Equation (35) can, by the same token, be put in a similar form:

$$\frac{d\alpha}{dt} = C_\rho{}^2 T^{-n}\frac{1}{\rho_D}\left\{-\alpha^2 + c^2\frac{1-\alpha^2}{1-c^2}\right\}$$

It follows from direct comparison that

$$C = 4k_R T^n(\rho_D/W_m^2)(1 + \alpha)$$

C also can be expressed in terms of the dissociation rate constant k_D cm³/mol-sec. At equilibrium, the relation between k_D and k_R is simply $k_D/k_R = K_C$, where K_C is the equilibrium constant. Again, using an ideal dissociated gas, one obtains the relation

$$K_C = (4\rho_D/W_m)e^{-D/RT}$$

and thus

$$C = k_D T^n e^{D/RT}(1/W_m)(1 + \alpha) \tag{37}$$

Matthews[12] gives dissociation rate constants for oxygen obtained from interferometric measurement in a shock tube.

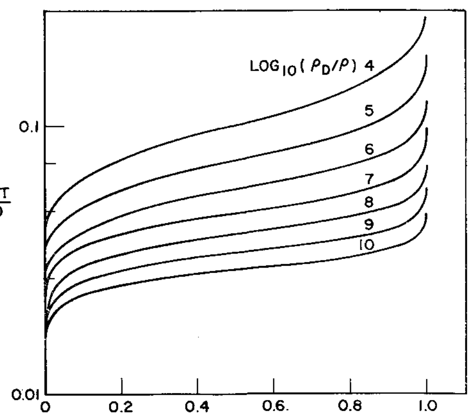

Fig. 2 Equilibrium mass fraction C

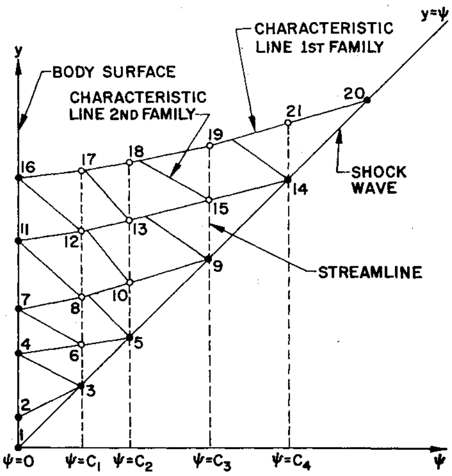

Fig. 3 Geometry of the numerical scheme

The mathematical fit of Matthews' data is as follows:

$$k_D = 7.43 \times 10^{11} \times 0.073\ T^{1/2}(59380/T)^3 e^{-59380/T}$$

59380 is the value of D/R in Eq. (37); C then can be evaluated numerically as

$$C = 7.43 \times 10^{11} \times 0.073 \times \tfrac{1}{32} \times (1 + \alpha) \times (59380)^3 \tag{38}$$

and the exponent n of T thus is equal to 2.5. Equation (38) indicates that C is not strictly a constant because of the $(1 + \alpha)$ multiplication factor affecting it. However, Matthews indicates a certain degree of uncertainty in the determination of the factor 0.073 and estimates the error to lie between the bounds of 0.05 and 0.1. The assumption involved in taking C as a constant is, therefore, within the experimental error of the evaluation of the dissociation rate. Similar data fit for the nitrogen dissociation rate are reported by Hammerling et al.[13] The exponent n of T is, in this case, equal to 1.5, and the numerical value of C is clearly different; the same $(1 + \alpha)$ dependency exists, however.

Using the expressions for the two equations of state (7) and (7′), it is now possible to evaluate the coefficients A, B, and F introduced earlier:

$$dp = \left(\frac{\partial p}{\partial e}\right)_{\rho,\alpha} de + \left(\frac{\partial p}{\partial \rho}\right)_{e,\alpha} d\rho + \left(\frac{\partial p}{\partial \alpha}\right)_{\rho,e} d\alpha$$

$$A = \left(\frac{\partial p}{\partial \rho}\right)_{S,\alpha} = \left(\frac{\partial p}{\partial e}\right)_{\rho,\alpha}\left(\frac{de}{d\rho}\right)_{S,\alpha} + \left(\frac{\partial p}{\partial \rho}\right)_{e,\alpha} + \left(\frac{\partial p}{\partial \alpha}\right)_{\rho,e}\left(\frac{d\alpha}{d\rho}\right)_{S,\alpha}$$

Using Eqs. (33–34′) and the second law of thermodynamics, Eq. (8), it follows by direct substitution that

$$a_f^2 = A = \left(\frac{dp}{d\rho}\right)_{S,\alpha} = RT\left\{\frac{(1+\alpha)^2}{3} + (1+\alpha)\right\}$$

Similarly, evaluating B and F by the same technique, it follows that

$$(\partial p/\partial S)_{\rho,\alpha} = B = (\rho T/3)(1 + \alpha)$$

and

$$(\partial p/\partial \alpha)_{\rho,S} = F = (\rho/3)(1+\alpha)(\mu_1 - \mu_2) + \rho\{RT - (D/3)(1+\alpha)\}$$

The difference between the chemical potential is then

$$\mu_1 - \mu_2 = -RT\ln\left(\frac{\rho_D}{\rho}e^{-D/RT}\frac{1-\alpha}{\alpha^2}\right)$$

by virtue of Eq. (32).

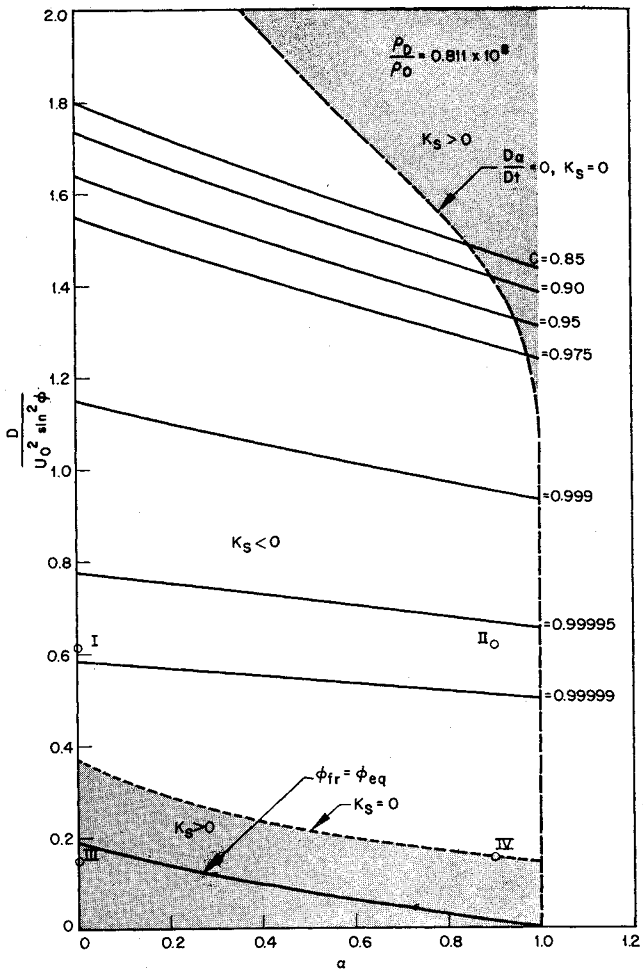

Fig. 4 Regimes of shock curvature

The characteristics Eq. (24) takes on a simpler form after substitution, namely,

$$d\xi \pm \frac{(M_f^2 - 1)^{1/2}}{\rho V^2}dp + \frac{D/3\,(1+\alpha) - RT}{a_f^2\,\rho V^2}\frac{\chi}{\theta}d\psi = 0 \tag{39}$$

The shock boundary conditions now can be solved, and one obtains

$$\begin{aligned} p_1 &= \frac{6\rho_0 V_0^2 \sin^2\varphi - (1+\alpha_0)p_0}{7+\alpha_0} \\ \rho_1 &= \frac{(7+\alpha_0)\rho_0^2 V_0^2 \sin^2\varphi}{2(4+\alpha_0)p_0 + (1+\alpha_0)\rho_0 V_0^2 \sin^2\varphi} \\ V_{N_1} &= \frac{2(4+\alpha_0)}{7+\alpha_0}\frac{p_0}{\rho_0 V_0 \sin\varphi} + \frac{1+\alpha_0}{7+\alpha_0}V_0\sin\varphi \\ \xi &= \varphi - \tan^{-1}\frac{V_{N_1}}{V_0\cos\varphi} \end{aligned} \tag{40}$$

Similarly, the entropy difference can be computed by integration of Eq. (8):

$$\begin{aligned}(S - S_0)/R = {} & 3\log(T/T_0) - (1+\alpha_0)\log(\rho/\rho_0) + \\ & (\alpha - \alpha_0)\log(\rho_D/\rho) + \{(1+\alpha) - \\ & (1-\alpha)\log(1-\alpha) - 2\alpha\log\alpha\} - \\ & \{(1+\alpha_0) - (1-\alpha_0)\log(1-\alpha_0) - 2\alpha_0\log\alpha_0\}\end{aligned} \tag{41}$$

Method of Solution

The advantages of writing the equations of motion in the (ψ, y) coordinate system become obvious when the numerical

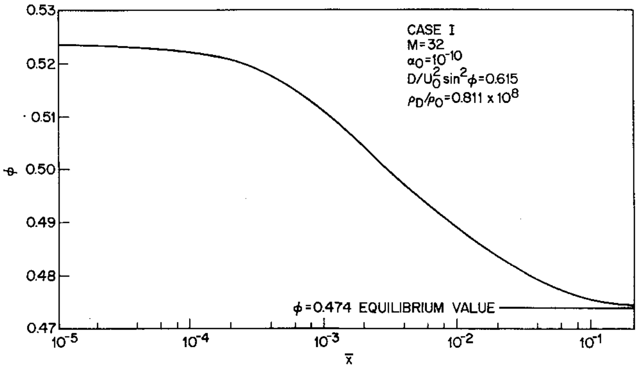

Fig. 5 Variation of shock wave angle

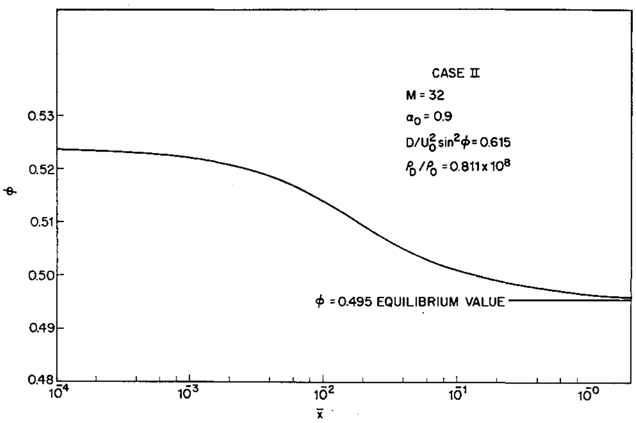

Fig. 6 Variation of shock wave angle

procedure of the method of characteristics is applied. Full use can be made of the third characteristic, i.e., the streamline, and, in general, it is possible to trace the streamlines throughout the flow field. In such a plane, the streamlines are the lines ψ = const, and the shock is a straight line (Fig. 3). Although the remaining two characteristics generally will not intersect on a ψ mesh point, the values of pressure and flow angle to the nearest ψ mesh point can be interpolated. The other flow properties such as S, α, and T can be computed using their stream derivatives.

To start the procedure, the flow properties at the tip of the wedge, using the initial frozen shock angle, are determined. Then, computing the initial pressure gradient along the wedge by Eq. (29), the flow values are determined at a neighboring point (an arbitrarily small distance along the wedge). The size of this initial step determines the mesh size for the subsequent procedure, and it must be adjusted for maximum convergence in the resulting flow field pattern. These initial values are sufficient to build, point by point, the entire flow field.

Discussion and Results

The fluid motion is determined by the geometry of the wedge, defined by its half-angle ξ_w, the freestream conditions, and the determining constants entering in the chemical rate equation, such as C, ρ, and D. By combining these dimensional constants in nondimensional groups, it is found that a particular problem is determined fully by the following system of dimensionless parameters: M_0, V_0^2/D, ρ_D/ρ_0, ξ_w, α_0. Subscript 0 refers to the freestream conditions; M_0 is the freestream Mach number; u_0^2/D is a measure of the kinetic energy contained in the freestream relative to the gas dissociation energy; ρ_D/ρ_0 is a measure of the freestream density; α_0 is the degree of dissociation of the oncoming freestream. However, the freestream fluid does not undergo any dissociation or recombination process. This freestream fluid may have assumed its present α_0 value either because its pressure and temperature call for an equilibrium value, or because the chemical reaction froze at this α_0 value in the course of, for example, an expansion process having originated at some upstream station.

The independent and dependent variables are nondimensionalized and are designated by their barred symbols as follows:

$$\bar{p} = p/\rho_0 V_0^2 \qquad \bar{\rho} = \rho/\rho_0$$
$$\bar{h} = h/V_0^2 \qquad \bar{T} = RT/V_0^2$$
$$\bar{S} = (S - S_0)/R \qquad \bar{x} = x\rho_0 R^n C/(V_0^{1+2n})$$
$$\bar{y} = y_0 \rho R^n C/(V_0^{1+2n}) \qquad \bar{\psi} = \psi/p_0 V_0 y$$

The nondimensionalization of x and y is equivalent to an equal stretching of both coordinates proportional to a length equal to $V_0^{1+2n}/\rho_0 R^n C$. This length is not related directly to the normally defined relaxation length. The latter would be equal to $V_0(\rho_D/\rho_0^2)(T_0^n/C)$ if Freeman's rate equation were used or to $V_0 W_m^2/k_R \rho_0^2$ if the more general rare equation, given by Clarke, were used. The nondimensionalization of ψ renders its value at the shock wave equal to the corresponding y value; therefore, in the y, ψ coordinate system, the shock wave is a straight line inclined at an angle of 45°.

For illustrative purposes, several solutions are presented. For all solutions, the Mach number M_0 was held fixed at a hypersonic value of $M_0 = 32$. Similarly, ρ_D/ρ_0 also was held constant at the value $\rho_D/\rho_0 = 0.811 \times 10^8$ which, for oxygen, represents an ambient altitude of 150,000 ft. The initial curvature of the shock wave, K_S, at the tip of the wedge, Eq. (30), is a helpful guide in selecting the values of u_0^2/D and α_0 which bring out the interesting features of the flow field. The sign of K_S depends only on the sign of the expression $(BJ\ \chi/\theta + F\chi/\theta)$ appearing in the numerator of Eq. (30). This expression can be rewritten, for the ideal dissociating gas and Freeman's rate equation, as

$$RT - (D/3)(1 + \alpha_0)(D_\alpha/Dt)$$

where T is evaluated at the wedge tip immediately behind the shock. $D\alpha/Dt$ is the convective derivative, also pertaining to the wedge surface and evaluated at the wedge tip. This expression can be evaluated, in terms of the freestream conditions and initial shock angle, as

$$\left\{\left(\frac{6 \sin^2\varphi - (1 + \alpha_0)(p_0/\rho_0 V_0^2)}{(7 + \alpha_0)^2 \sin^2\varphi}\right) \times \left(\frac{2(4 + \alpha_0)(p_0/p_0 V_0^2) + (1 + \alpha_0) \sin^2\varphi}{1 + \alpha_0}\right) - \frac{D}{3V_0^2}(1 + \alpha_0)\right\} \frac{D\alpha}{Dt} V_0^2$$

For hypersonic Mach numbers, $1/M_0^2 \ll 1$, and with the added restriction that $1/M_0^2 \ll D/V_0^2$, the foregoing ex-

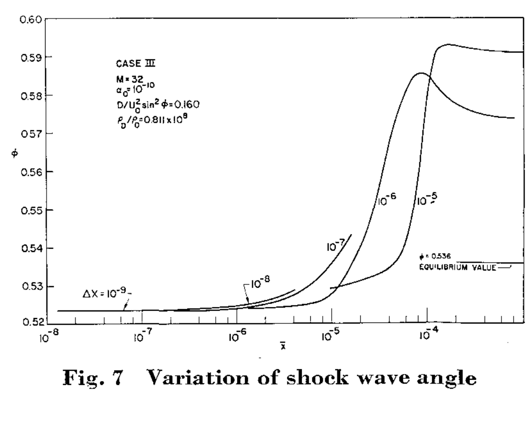

Fig. 7 Variation of shock wave angle

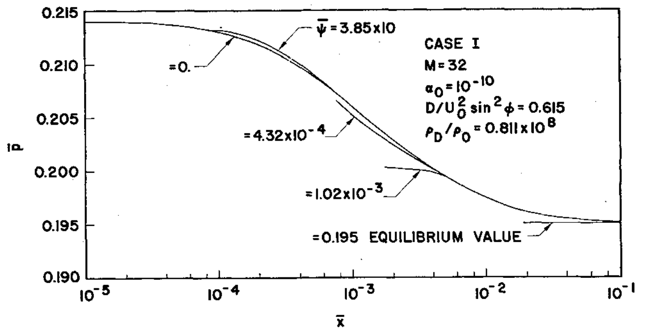

Fig. 8 Pressure distribution along streamlines

pression can be simplified. As a result, K_S will have the sign of

$$\left\{\frac{6}{(7+\alpha_0)^2} - \frac{D}{3V_0^2\sin^2\varphi}\right\}(1+\alpha_0)\frac{D\alpha}{Dt} \tag{42}$$

Clearly, the shock wave curvature is zero when $D\alpha/Dt = 0$. This corresponds to the case in which the equilibrium atomic mass fraction c at the wedge tip behind the shock is precisely equal to α_0. The shock wave has brought the overdissociated freestream exactly into equilibrium. The flow properties are then constant throughout the flow field, since no further chemical reaction takes place. The expression in braces in Eq. (42) can, however, be equal to zero independently of $D\alpha/Dt$. The roots of this expression coincide with those of a cubic equation in α_0. There always exists one real root between 0 and 1 as long as the parameter $D/V_0^2 \sin^2\varphi$ varies between the values $\frac{18}{49}$ and $\frac{18}{128}$.

In such cases, the shock wave is locally straight near the wedge tip, although the flow properties behind it are not constant as a chemical reaction occurs. Figure 4 presents graphically the variation of α_0 vs $D/V_0^2 \sin^2\varphi$ corresponding to the lower curve labeled $K_S = 0$. The portion above the curve corresponds to a negative value of the bracketed expression, whereas the portion below the curve yields a positive value. The sign of the curvature of the shock can be determined only if the sign of the $D\alpha/Dt$ also is known. For this purpose, the curves of the equilibrium mass fraction c have been superimposed on the forementioned graph. The c values can be obtained simply by solving Eq. (32) for known values of α_0 and $D/V_0^2 \sin^2\varphi$. If c is larger than α_0, then $D\alpha/Dt$ is positive, and conversely. In this fashion, regions can be defined in which the shock wave initially will be concave ($K_S > 0$) or convex ($K_S < 0$).

Further examination of the expression $(BJ\chi/\theta + F\chi/\theta)$ reveals that it can be rewritten as

$$(D\alpha/Dt)(\partial p/\partial\alpha)_{\rho,S} + (DS/Dt)(\partial p/\partial S)_{\rho,\alpha}$$

A linearized treatment of the flow field would eliminate the term DS/Dt, the entropy production term, which is of second order, and K_S depends consequently only on the sign of $D\alpha/Dt$. The shock wave is convex for dissociating flows $D\alpha/Dt > 0$ or concave for recombining flows $D\alpha/Dt < 0$; this agrees with the findings of Vincenti.[7] Retaining the DS/Dt term complicates the picture somewhat, as is evident from the foregoing discussion. Of course, it might be well to investigate whether or not the choice of the equation of state changes the results of this discussion qualitatively. After some rearranging, expression (42) assumes the form

$$(\partial p/\partial\alpha)_{\rho,e}(D\alpha/Dt)$$

The internal energy e can be expressed conveniently as the sum of two terms: a "sensible" internal energy e_s, and the internal energy of chemical bonding e_c; e_c is generally taken as $e_c = D\alpha$, where D is the dissociation energy; e_s is related

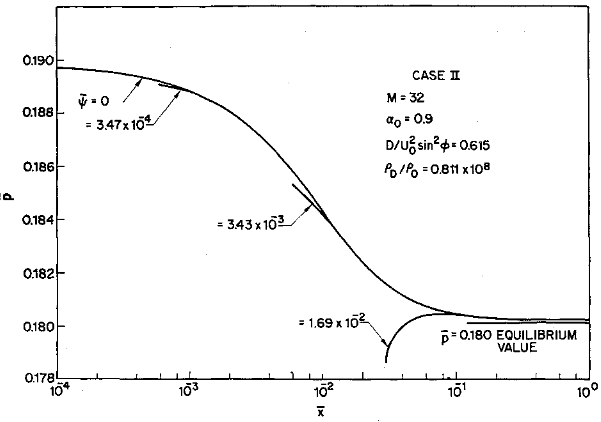

Fig. 9 Pressure distribution along streamlines

to p, α, and ρ by an equation of state; in the simpler case, e_s is expressed by a perfect gas equation of state. Thus in general, the energy has the following form:

$$e = f(p,\rho,\alpha) + D\alpha$$

Consequently, $(\partial p/\partial\alpha)_{\rho,e}$ is a sum of two terms and may change its sign around a possible zero point.

The asymptotic value of the shock wave angle is equal, as Sedney shows in Ref. 9, to the value of the equilibrium shock angle. This value can be computed from the shock equation (31), in which h_0 and h_1 are given by $h_0 = (4 + \alpha_0)RT_0 + D\alpha_0$ and $h_1 = (4 + c_1)RT_1 + Dc_1$. The equilibrium concentration c_1 is related to T_1 and ρ_1 by Eq. (32), which gives the equilibrium mass fraction c_1 at the local equilibrium density and temperature. A series of trials and errors, in general, is necessary to obtain the shock equilibrium conditions. The locus of the points where the frozen shock angle φ_{fr} and the equilibrium shock angle φ_{eq} are equal to one another is plotted in Fig. 4 and labeled as $\varphi_{fr} = \varphi_{eq}$. Under the hypersonic approximation $1/M_0^2 \ll 1$ introduced earlier, this curve can be determined independently of the values of the wedge angle and necessitates only knowledge of the parameters $D/V_0^2 \sin^2\varphi$, α_0, and ρ_D/ρ_0.

Similarly, it is proven easily that, for value of $D/V_0^2 \sin^2\varphi$ lying above the curve $\varphi_{fr} = \varphi_{eq}$, the value of φ_{fr} is larger than φ_{eq}, whereas the converse is true for value of $D/V_0^2\sin^2\varphi$ lying below this curve. This result also is independent of the value of the wedge angle. In the area contained between the curves labeled $K_S = 0$ and $\varphi_{fr} = \varphi_{eq}$, the shocks will assume an "S" shape, since they are initially concave but tend asymptotically toward a shock angle value that is smaller than the one at the tip of the wedge. A similar argument could be presented for values of $D/V_0^2 \sin^2\varphi = 2.0$ and above. In this area, however, the hypersonic approximation is not valid, and the degree of dissociation is small.

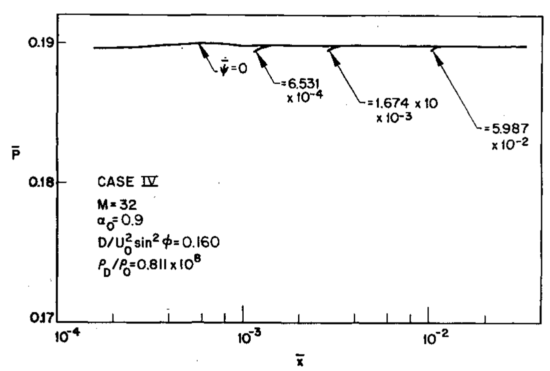

Fig. 10 Pressure distribution along streamlines

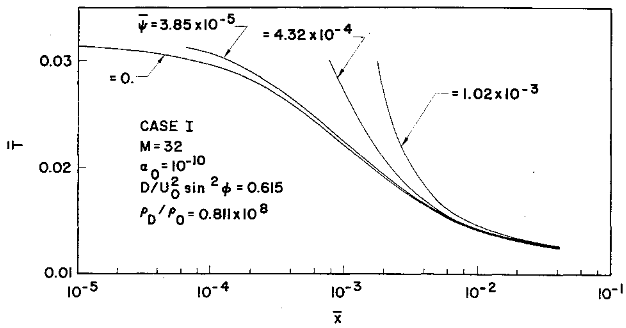

Fig. 11 Temperature distribution along streamlines

In order to illustrate better the foregoing discussion, four cases have been selected and are represented as follows:

Case I:

$$D/V_0^2 \sin^2\varphi = 0.615$$
$$\alpha_0 = 10^{-10}$$

Case II:

$$D/V_0^2 \sin^2\varphi = 0.615$$
$$\alpha_0 = 0.9$$

Case III:

$$D/V_0^2 \sin^2\varphi = 0.160$$
$$\alpha_0 = 10^{-10}$$

Case IV:

$$D/V_0^2 \sin^2\varphi = 0.160$$
$$\alpha_0 = 0.9$$

The four cases are indicated by their numbers in Fig. 4. As can be seen from the graph, all cases will correspond to a high degree of dissociation behind the shock wave. Cases I and II will correspond to a convex shock wave and III to a concave one. Case IV was chosen purposely to yield an initial shock wave curvature equal to zero. As shown in the Appendix, the shock is only initially straight and cannot remain so indefinitely. Rather than selecting the wedge angle, an initial shock angle of 30° was chosen for all cases. The variation of the shock wave angle with $\bar{x}$ is plotted for cases I, II, and III in Figs. 5–7. Case IV yielded a practically straight shock wave, the initial angle being equal to 0.5236 rad (30°) and leveling off to an equilibrium value of 0.5233 rad.

The equilibrium shock angle values φ_{eq} have been indicated. In case I, the shock angle has not reached its equilibrium value but is approaching it at such a slow rate that the computation was interrupted before completion.

Case III is an example of the difficulties experienced in the integration of the rate equation when the initial value of the rate of dissociation $d\alpha/dt$ is very large. At high temperatures, the relaxation time for dissociation is extremely small, being proportional to k_D^{-1}, which varies as seen from Eqs. (37) and (38) as $T^{2.5}e^{D/RT}$. The numerator χ of the generalized rate equation, Eq. (6), assumes also very large values, since it is proportional to $(c^2 - \alpha^2)/(1 - c^2)$, and since for high temperatures c tends towards $1 - \rho/\rho_D$. For case III, ρ/ρ_D was chosen to be approximately 10^{-8}. To insure convergence to the proper values of the physical flow quantities, extremely small steps of integration must be taken. This is shown in Fig. 7, where Δx refers to the initial step size along the wedge surface. Too large a value of Δx will give an erroneous shock angle variation. A step size as small as 10^{-7} still yields a shock angle that overshoots the equilibrium value. For $\Delta x = 10^{-8}$, it would appear that

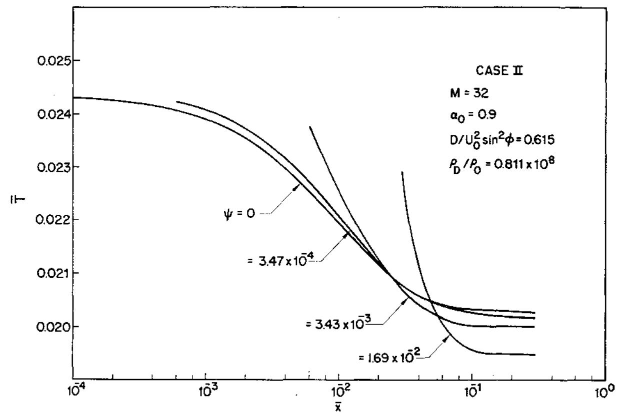

Fig. 12 Temperature distribution along streamlines

the proper variation is attained. Because of the unduly lengthy time of integration, the computation was not carried out any further than shown.

The phenomenon described here is not particular to this problem but is related to the general behavior of the rate equation at high temperatures and for values of α far removed from equilibrium. Because of the strong nonlinear character of the rate equation for this set of circumstances, a "sublayer" exists inbedded in the relaxation layer within which rapid changes in temperature, degree of dissociation, and relaxation times will take place. A special stretching of the coordinate system is necessary to examine the details of this sublayer.

The influence of the chemical reaction on the flow properties is understood easily in light of the characteristic equation, Eq. (24). The first two terms $d\xi$ and $[(M_f^2 - 1)^{1/2}/\rho V^2]dp$ are the familiar terms entering in the two-dimensional characteristics equation for nonreacting or equilibrium flows. The third term, on the other hand, represents the contribution of the reactions. Within the third term, $F(\chi/\theta)(d\psi/a_f^2\rho^2V^2)$ represents the effect of the change of composition of the mixture, whereas $BJ(\chi/\theta)(d\psi/a_f^2\rho^2V^2)$ accounts for the entropy production between two neighboring points on the streamline. Both terms contribute to the production of pressure waves, the first because of production or disappearance of atoms and the resulting effect of heat subtraction or addition and the second because of the heat addition due to the irreversibility of the process.

Indeed, the two expressions can be rewritten (by virtue of the definition of the symbols) as follows:

$$F\frac{\chi}{\theta}\frac{d\psi}{a_f^2\rho^2V^2} = \left(\frac{\partial p}{\partial \alpha}\right)_{\rho,S}\frac{D\alpha}{Dt}\frac{d\psi}{a_f^2\rho^2V^2}$$

$$BJ\frac{\chi}{\theta}\frac{d\psi}{a_f^2\rho^2V^2} = \left(\frac{\partial p}{\partial S}\right)_{\rho,\alpha}\left(-\frac{\partial S}{\partial \alpha}\right)_T\frac{D\alpha}{Dt}\frac{d\psi}{a_f^2\rho^2V^2}$$

These two terms counteract one another, and, for certain

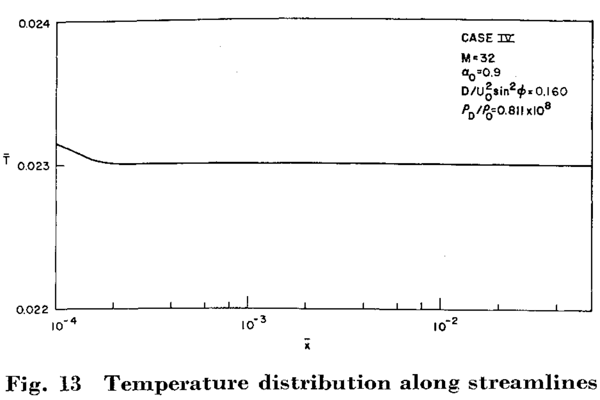

Fig. 13 Temperature distribution along streamlines

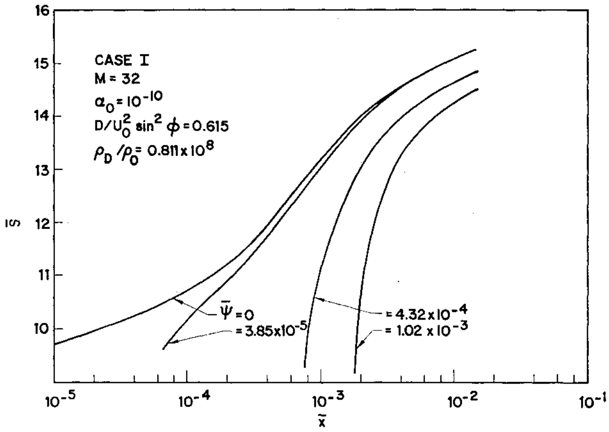

Fig. 14 Entropy distribution along streamlines

cases, as seen in the foregoing expressions, their influence is counterbalanced exactly, leading, consequently, to a zero net change in pressure. These pressure signals are propagated along the frozen Mach lines. For the case of equal and opposite pressure wave signals, the pressure should remain constant along the frozen Mach lines, at least locally. In general, however, the net result is different from zero, and an increase or decrease of pressure occurs, depending on which term dominates. Upon interaction with the shock wave, an increase in shock angle (concave shock) or decrease in shock angle (convex shock) will result, brought about by either a pressure increase signal or a pressure decrease signal. This is illustrated in Figs. 8–10, showing the pressure distributions along several typical streamlines for the three cases. The corresponding shock waves were shown earlier. Correspondingly, the temperature distributions are shown in Figs. 11–13, the entropy distribution in Figs. 14–16, and the α distributions in Figs. 17–19.

All of the forementioned curves illustrate rapid changes in the physical flow properties near the shock. These changes are concentrated within a thin layer called the relaxation layer. Outside this layer, the flow properties change more slowly as they approach equilibrium. It will be noticed that, even though the shock wave almost has attained its equilibrium angle in all four cases, the flow properties have not leveled off to an equilibrium value in cases I and II. This can be explained by the fact that, in the present mesh construction scheme, the shock point always lies a relatively large distance ahead of the corresponding body point located on the same characteristic line. To obtain points on the body located at larger distances would necessitate carrying on the integration for undue lengths of time.

Figures 20–22 illustrate, for a given $\bar{x}$, the distribution of the flow properties with respect to $\bar{y}$, the lateral coordinate extending from the body to the shock. The relaxation layer

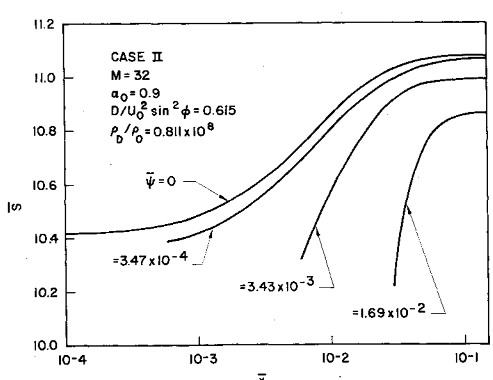

Fig. 15 Entropy distribution along streamlines

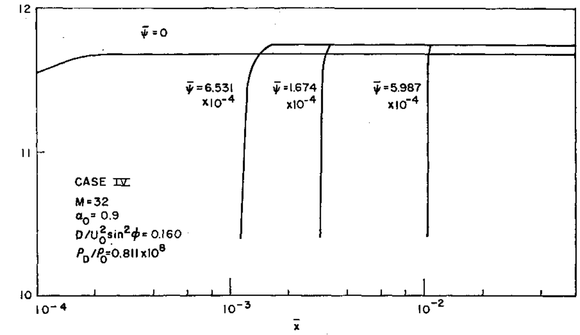

Fig. 16 Entropy distribution along streamlines

in this case is traversed vertically, as shown by the same rapid changes of properties near the shock surface.

The physical dimensions of the flow can be obtained easily by selecting particular freestream conditions and the determining constants of the chemical rate equation. Of particular interest is the length of the relaxation process. At a ρ/ρ_D of 0.811×10^8 which corresponds to an altitude of 150,000 ft, the value of V_0 for cases I and II is approximately 30,000 fps, whereas it is 60,000 fps for cases III and IV. The value of C can be taken from Eq. (38) as a result of Matthews'[12] experiments for oxygen dissociation. A value of $\bar{x} = 2 \times 10^{-4}$ for the relaxation length on the body in case III

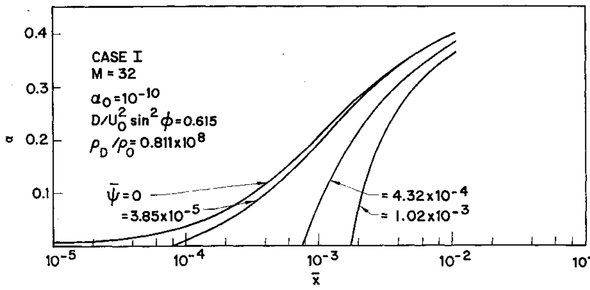

Fig. 17 Degree of dissociation along streamlines

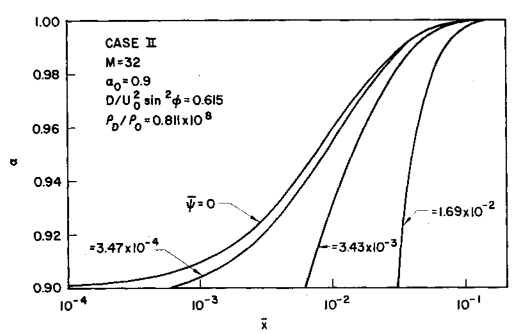

Fig. 18 Degree of dissociation along streamlines

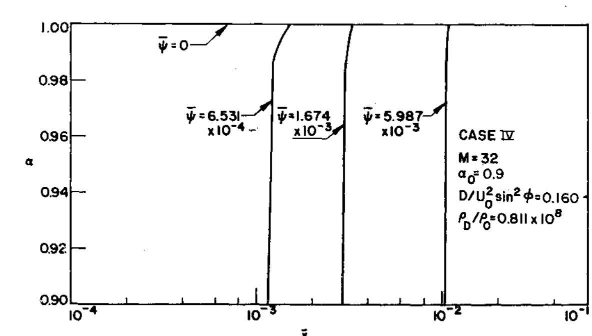

Fig. 19 Degree of dissociation along streamlines

corresponds to a value of x equal to 0.6 cm. Similarly, for case II, a value of x equal to 10 cm is obtained. The mean free path at this altitude is about 4×10^{-3} cm, thus corresponding to 150 collisions for complete dissociation in case III and 2500 collisions in case II.

Conclusions

The solution of the flow field of a chemically reacting gas past a wedge has been presented. The different regimes of shock concavity or convexity have been delineated, and numerical examples have been carried out. The relaxation layer and entropy layer have been identified for the cases studied. Future work will bear on a study of the finer structure of both these layers. Experimental data of shock layer structure for chemically reacting flows will be used to examine critically the validity of the physical assumptions that entered the rate equation and the model for the ideal dissociating gas.

Appendix

In the preceding discussion, it was found that the shock wave will be initially straight under two sets of circumstances: 1) when the overdissociated freestream is put in equilibrium immediately behind the shock as a result of the temperature and density rise associated with the shock; and 2) when the quantity $RT - (D/3)(1 + \alpha)$ is equal to zero, or, more generally, when $BJ\chi/\theta + F\chi/\theta$ is zero.

In order to determine when a straight shock wave will generate a straight body, assume that the shock is straight and derive necessary and sufficient conditions for a straight body.

It is more convenient to rotate the x, y coordinate system to a (η, σ) system so that the shock becomes one of the coordinates: $\eta = 0$ (Fig. 23). It follows from geometry that $\eta(\partial/\partial\sigma) = 0$, and the equations of motion (1–3, 6, and 9) can be written as follows:

$$(d/d\eta)(\rho u \sin\varphi - \rho v \cos\varphi) = 0$$

$$(\rho u \sin\varphi - \rho v \cos\varphi)(du/d\eta) + \sin\varphi(dp/d\eta) = 0$$

$$(\rho u \sin\varphi - \rho v \cos\varphi)(dv/d\eta) - \cos\varphi(dp/d\eta) = 0 \quad (43)$$

$$(u \sin\varphi - v \cos\varphi)(d\alpha/d\eta) = \chi/\theta$$

$$(u \sin\varphi - v \cos\varphi)(dS/d\eta) = J\chi/\theta$$

The first of these equations is integrated and gives

$$\rho u \sin\varphi - \rho v \cos\varphi = \text{const} = K$$

Differentiating the equation of state (7) with respect to η, it follows that

$$\frac{dp}{d\eta} = a_f^2 \frac{d\rho}{d\eta} + \left(BJ\frac{\chi}{\theta} + F\frac{\chi}{\theta}\right)\frac{\rho}{K}\frac{d\alpha}{d\eta}$$

By elimination, the following three equations are obtained:

$$du/d\eta = -\tan\varphi(dv/d\eta)$$

$$\frac{du}{d\eta}\left(\rho^2\frac{a_f^2}{K} - K\right)\frac{1}{\sin\varphi} = \frac{\rho}{K}\left(BJ\frac{\chi}{\theta} + F\frac{\chi}{\theta}\right)\frac{d\alpha}{d\eta}$$

$$d\alpha/d\eta = \chi/\theta$$

A forward integration now could be performed starting from known conditions at the shock wave; thus u and v would be obtained, yielding a body geometry. However, the initial values of u and v which determine the initial body angle are known from the shock conditions at $\eta = \sigma = 0$.

In order to obtain a straight wedge, it follows from direct examination of the first two equations of Eq. (43) that $du/d\eta$ and $dv/d\eta$ must always remain zero. This can happen only if $d\alpha/d\eta$ or $(BJ\chi/\theta + F\chi/\theta)$ are zero throughout the flow field.

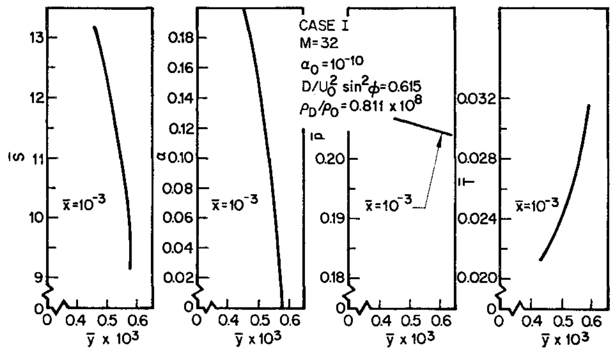

Fig. 20 Variation of flow properties at constant $\bar{x}$

If $d\alpha/d\eta$ is initially zero at the shock wave surface, the foregoing equations imply that it will remain zero indefinitely. Once the gas is in equilibrium, it remains in equilibrium.

If, on the other hand, $BJ(\chi/\theta) + F(\chi/\theta)$ is initially zero, it cannot remain equal to zero. Taking the derivative with respect to η of the expression, it follows that $(d/d\eta)(RT) - (D/3)(d\alpha/d\eta)$ should be equal to zero, at least at the shock wave front. This, however, is in contradiction with the energy equation $h + (V^2/2) = $ const which can be written in its differential form as

$$\frac{d}{d\eta}(RT)\frac{1}{RT}(4 + \alpha) + \frac{d\alpha}{d\eta}\left(\frac{D}{RT} + 1\right) - (1 + \alpha)\frac{dp}{d\eta}\frac{1}{p} = 0$$

Since $dp/d\eta = 0$ for a straight shock, this reduces to

$$(d/d\eta)(RT)(4 + \alpha) + (d\alpha/d\eta)(D + RT) = 0$$

which is irreconcilable with the condition that $(d/d\eta)(RT) - (D/3)(d\alpha/d\eta)$ also be equal to zero.

The necessary condition for obtaining a straight shock is, therefore, that equilibrium condition $D\alpha/Dt = 0$ be reached immediately behind it.

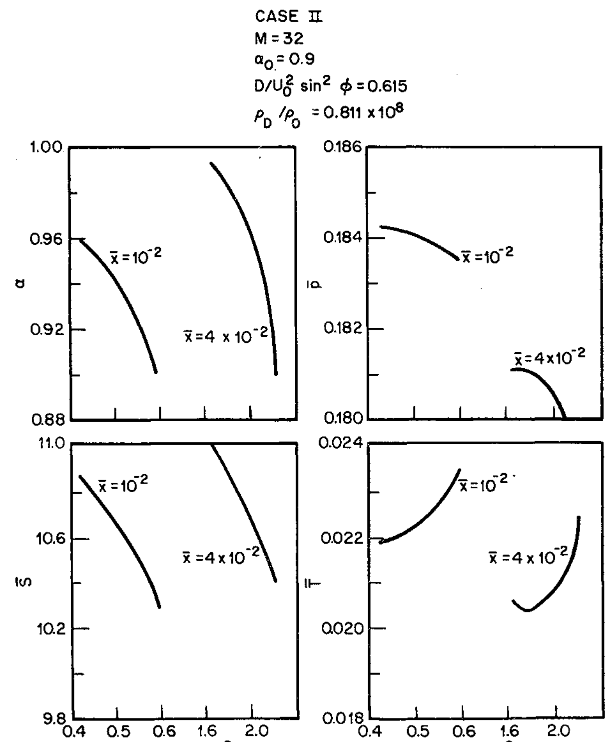

Fig. 21 Variation of flow properties at constant $\bar{x}$

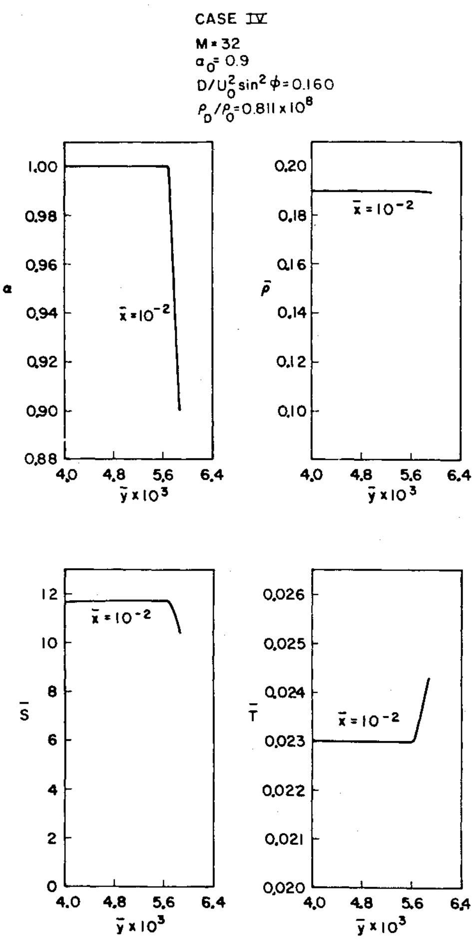

Fig. 22 Variation of flow properties at constant $\bar{x}$

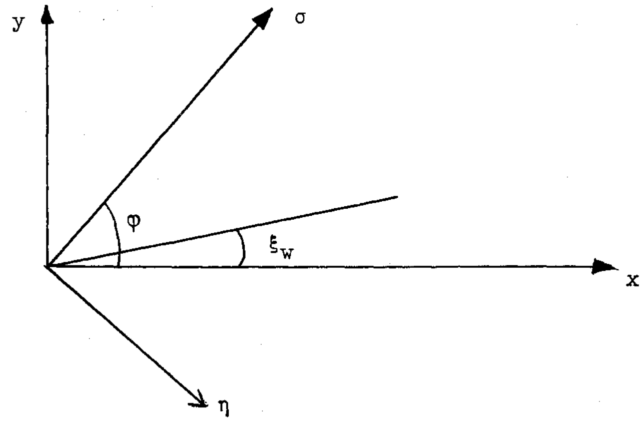

Fig. 23 Shockwave oriented coordinate system

References

[1] Wood, W. W. and Kirkwood, J. G., "Hydrodynamics of a reacting and relaxing fluid," J. Appl. Phys. **28**, 395–398 (1957).

[2] Chu, B. T., "Wave propagation and the method of characteristics in reacting gas mixtures with application to hypersonic flow," Wright Air Dev. Center TN 57-213 (1957).

[3] Clarke, J. F., "The linearized flow of a dissociating gas," J. Mech. **7**, 577–59 (1960).

[4] Sedney, R., South, J. C., and Gerber, N., "Characteristics calculation of nonequilibrium flows," Ballistics Research Lab., Aberdeen, Md., BRL Rept. 1173 (1962).

[5] Lighthill, M. J., "Dynamics of a dissociating gas, Part I, Equilibrium flow," J. Fluid Mech. **2**, 1–32 (1957).

[6] Freeman, N. C., "Dynamics of a dissociating gas, nonequilibrium theory," High Altitude and Satellite Rockets Symposium Reprint (1957).

[7] Vincenti, W. G., "Linearized flow over a wedge in a nonequilibrium oncoming stream," Stanford Univ. Rept. SUDAER 123 (1962).

[8] Clarke, J. F., "The flow of chemically reacting gas mixtures," College of Aeronautics, Cranfield, England, Rept. 117 (1958).

[9] Sedney, R., "Some aspects of nonequilibrium flows," J. Aerospace Sci. **28**, 189–196 (1961).

[10] Hsu, C. T., "On the gradient functions for nonequilibrium dissociation flow behind a shock wave," J. Aerospace Sci. **28**, 337–339 (1961).

[11] Clarke, J. F., "Energy transfer through a dissociated diatomic gas in couette flow," J. Fluid Mech. **4**, 441–465 (1958).

[12] Matthews, D. L., "Interferometric measurement in the shock tube of the dissociation rate of oxygen," Phys. Fluids **2**, 170–178 (1959).

[13] Hammerling, P., Teare, J. D., and Kivel, B., "Theory of radiation from luminous shock waves in nitrogen," Phys. Fluids **2**, 422–426 (1959).

CALL FOR PAPERS

Conference on Physics of Entry into Planetary Atmospheres

Sponsored by American Institute of Aeronautics and Astronautics and the Massachusetts Institute of Technology

KRESGE AUDITORIUM, MIT • AUGUST 26–28, 1963 • CAMBRIDGE, MASSACHUSETTS

The purpose of the conference is to assay the present state of knowledge on the flow environments and physical, chemical, and electrical phenomena that arise in connection with the entry of bodies at high velocity into the atmosphere of the earth and other planets of the solar system. Emphasis will be placed on the fundamental aspects of "Entry Physics" and not on specific design problems of atmospheric entry. In this regard, both experimental and theoretical papers are solicited, though papers dealing primarily with facilities are considered to be outside the scope of the conference. The conference will be divided into six consecutive sessions.

Session topics as presently defined are: Physics of High Temperature Gases • Gas-Surface Interaction • Hypersonic Flow • High Temperature Gas Flow • Early Entry Phenomena.

Authors should submit 2- or 3-page abstracts in duplicate before **May 15, 1963** to the conference chairman:

Professor Ronald F. Probstein
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts